Constraint on modified dispersion relations for gravitational waves from gravitational Cherenkov radiation

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We investigate the hypothetical process of gravitational Cherenkov radiation, which may occur in modified gravity theories. We obtain a useful constraint on a modified dispersion relation for propagating modes of gravitational waves, which could be predicted as a consequence of violation of the Lorentz invariance in modified theories of gravity. The constraint from gravitational Cherenkov radiation and that from direct measurements of the gravitational waves emitted by a compact binary system are complementary to each other.

Gravitational waves, predicted in gravitational theories, are at the frontier of physics and astronomy. Recently, a variety of the gravitational theories have been proposed, motivated by a possible explanation of the accelerated expansion of the universe. For example, in recent research viable gravitational theories of gravitational waves with a mass term have been proposed [1–3]. Furthermore, graviton oscillations are predicted in the ghost-free bigravity theory [4, 5]. In the most general scalar tensor theory of the second derivative [6], which was rediscovered recently [7], the propagation speed of gravitational waves may deviate from the propagation speed of light [8]. Within general relativity, the propagation speed of gravitational waves is the same as that of light, whose deviation is related to the breaking of Lorentz invariance [9, 10]. Thus, the gravitational wave is important to characterize modified theories of gravity.

The dispersion relation for gravitational waves has been much argued in the literature. For example, constraints on the mass of gravitational waves have been discussed [11]. Although gravitational waves have not been directly detected, the progress of observations such as those from the advanced LIGO project and the KAGRA project will make direct measurements possible. Assuming such future prospects of observational experiments, Mirshekari, Yunes, and Will investigated a future constraint on the modified dispersion relation through gravitational wave experiments [12]. They demonstrated that a stringent constraint on the mass term can be obtained with future direct measurements of gravitational waves emitted by a compact binary system. The authors of the recent papers [13, 14] further investigated the orbital evolution of binary pulsars in modified gravity models with Lorentz symmetry breaking and obtained a useful constraint on the model parameters.

However, it is known that gravitational Cherenkov radiation (GCR) arises if a massive particle moves faster than the speed of the gravitational waves, which is possible when the propagation speed of the gravitational waves is smaller than that of light [15]. This hypothetical process puts a stringent constraint on the propagation speed of gravitational waves by including observations of extremely high energy cosmic rays [16]. The usefulness is indeed demonstrated for modified gravity models of the new Ether-Einstein gravity and the Galileon-type cosmological model [17, 18].

In the present letter, we investigate the constraint on the modified dispersion relation for gravitational waves from GCR. We assume the same modified dispersion relation for the gravitational waves in Ref. [12]:

$$\omega_k^2 = k^2c_s^2 + m_g^2c_s^4 + Ak^\alpha c_s^\alpha, \tag{1}$$

where k, c_s, and m_g are the wave number, the propagation speed, and the mass of the gravitational wave, respectively, and A and α are also the parameters of the modified dispersion relation. In the absence of the terms of m_g, A, and α, it is known that GCR puts the following constraint on the propagation speed [16]:

$$1 - c_s \lesssim 2\times 10^{-17}\left(\frac{10^{11}\ \mathrm{GeV}}{p}\right)^{3/2}\left(\frac{1\ \mathrm{Mpc}}{ct}\right)^{1/2}. \tag{2}$$

However, the GCR process for the modified dispersion relation of the form (1) has not been discussed. We demonstrate that GCR puts a stringent constraint on A and c_s, depending on α and the sign of A, and that GCR cannot put a useful constraint on m_g. From the direct measurement of gravitational waves, the constraint on m_g is stringent, but the constraint on A is not very stringent [12], which means that the two methods are complementary.

Following Ref. [16, 18], the rate of emitting energy through GCR from a massive particle with mass m and relativistic momentum $p = |\mathbf{p}|$ can be written as

$$\frac{dE}{dt} = G_N p^2 \int_0^\infty dk k^2 \int_{-1}^{+1} d\cos\theta \sin^4\theta \times \delta_D(\Omega_i - \Omega_f - \omega_k), \tag{3}$$

where G_N is Newton's universal gravitational constant, Ω_i and Ω_f are defined by $\Omega_i = \sqrt{\mathbf{p}^2 + m^2}$ and $\Omega_f = \sqrt{(\mathbf{p}-\mathbf{k})^2 + m^2}$, respectively, and δ_D is the Dirac delta function. Using the identity $\delta_D(\Omega_i - \Omega_f -$

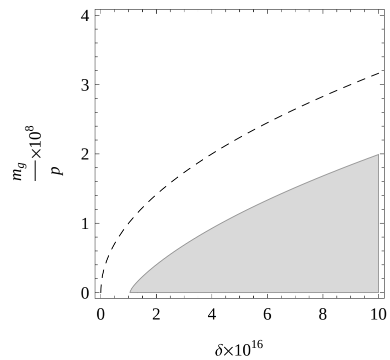

FIG. 1. Constraint on the model parameters from GCR for m_g/p and δ in the case $A = 0$. The shaded region satisfies (11); this region is excluded by GCR, where we assumed $p = 10^{11}$ GeV and $ct = 1$ Mpc. The dashed curve shows (8) with $k = p$, above which the GCR process does not appear effectively.

$\omega_k) = \ 2\Omega_f\delta_D(\Omega_f^2 - (\Omega_i-\omega_k)^2)$, we may write

$$\delta_D(\Omega_i-\Omega_f-\omega_k) = \frac{\Omega_f}{pk}\delta_D\Biggl(\cos\theta - \frac{k}{2p} + \frac{c_s^2 k}{2p}\left(1+\frac{c_s^2 m_g^2}{k^2}+A(c_s k)^{\alpha-2}\right) - \frac{c_s}{p}\sqrt{p^2+m^2}\sqrt{1+\frac{c_s^2 m_g^2}{k^2}+A(c_s k)^{\alpha-2}}\Biggr). \quad (4)$$

Integration of (3) makes a nontrivial contribution if

$$\cos\theta = \frac{k}{2p} - \frac{c_s^2 k}{2p}\left(1+\frac{c_s^2 m_g^2}{k^2}+A(c_s k)^{\alpha-2}\right) + \frac{c_s}{p}\sqrt{p^2+m^2}\sqrt{1+\frac{c_s^2 m_g^2}{k^2}+A(c_s k)^{\alpha-2}} \le 1, \quad (5)$$

which is a condition at which GCR arises. Assuming $m/p \ll 1$, $m_g/k \ll 1$, $A(c_s k)^{\alpha-2} \ll 1$, and $|\delta| \ll 1$, where we defined $\delta = 1 - c_s^2$, we have

$$\cos\theta \simeq 1 + \frac{(k-p)}{2p}\left(\delta - \frac{m_g^2}{k^2} - Ak^{\alpha-2}\right). \quad (6)$$

We may assume $k - p \le 0$, and then condition (5) leads to

$$\delta - \frac{m_g^2}{k^2} - Ak^{\alpha-2} \ge 0. \quad (7)$$

First we consider the case $A = 0$ but with finite graviton mass $m_g \neq 0$ and $c_s < 1$, i.e., $\delta > 0$. In this case, (7) gives

$$\delta - \frac{m_g^2}{k^2} \ge 0, \quad (8)$$

thus GCR does not appear when $\delta = 0$. We integrate Eq. (3) by adopting the approximation $\theta \ll 1$, and we have

$$\frac{dE}{dt} = G_N\int_{m_g/\sqrt{\delta}}^{k_{\max}} dkk\left((k-p)\left(\delta - \frac{m_g^2}{k^2}\right)\right)^2, \quad (9)$$

which yields

$$\frac{dE}{dt} = \frac{G_N}{12}\Biggl(27m_g^4 - 64\delta^{1/2}pm_g^3 + 36\delta p^2 m_g^2 + p^4\delta^2 - 12m_g^2(m_g^2-2p^2\delta)\ln\left[\frac{m_g}{p\delta^{1/2}}\right]\Biggr), \quad (10)$$

where we set $k_{\max} = p$. We estimate the condition at which the damping from GCR is significant for an extremely high energy cosmic ray with initial energy p during time t as $dE/dt > p/t$, which yields

$$27m_g^4 - 64\delta^{1/2}pm_g^3 + 36\delta p^2 m_g^2 + p^4\delta^2 - 12m_g^2(m_g^2-2p^2\delta)\ln\left[\frac{m_g}{p\delta^{1/2}}\right] > \frac{12}{G_N}\frac{p}{t}. \quad (11)$$

The shaded region in Fig. 1 satisfies this condition (11), which is excluded as the constraint of GCR. Here we assume an extremely high energy cosmic ray of $p = 10^{11}$ GeV from a cosmological distance $ct = 1$ Mpc. The dominant contribution of the k integration in Eq. (9) comes from the region of k of the order of p. The dashed curve in Fig. 1 shows the border of (8) with k replaced by p, i.e., $\delta = m_g^2/p^2$. In the region above the dashed curve, GCR does not occur effectively. Thus we obtain the constraint from GCR process for $\delta \gtrsim 10^{-16}$; however, the constraint on m_g is of the order of $m_g \lesssim 10^3$ GeV. When a high energy cosmic ray of $p = 10^3$ GeV from a cosmological distance $ct = 1$Mpc is assumed, we obtain the constraint on the m_g is of the order of $m_g \lesssim 10$ GeV for $\delta \gtrsim 10^{-4}$. Thus the constraint on the graviton mass from GCR is not very stringent.

Next we consider the case $m_g = 0$; the condition at which GCR arises, (7), is

$$\delta - Ak^{\alpha-2} \ge 0. \quad (12)$$

One can observe that GCR arises even when $\delta < 0$, i.e., $c_s > 1$, if A is negative. This is quite a contrast to the usual conclusion that GCR arises only when $c_s < 1$ [15, 16]. We integrate Eq. (3), adopting the approximation $\theta \ll 1$, which gives

$$\frac{dE}{dt} = G_N\int_0^p dkk\left((k-p)\left(\delta - Ak^{\alpha-2}\right)\right)^2, = \frac{G_N}{12}\left(\delta^2p^4 + \frac{6A^2p^{2\alpha}}{\alpha(\alpha-1)(2\alpha-1)} - \frac{48\delta Ap^{\alpha+2}}{\alpha(\alpha+1)(\alpha+2)}\right). \quad (13)$$

We here estimate the condition at which no significant radiation damping occurs from GCR for an extremely

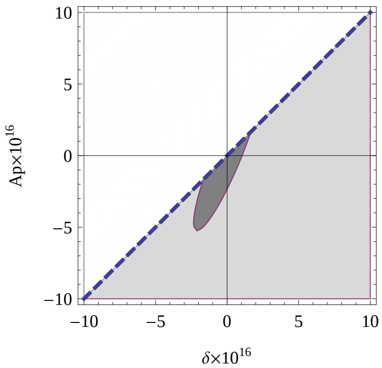

FIG. 2. Constraint on the model parameters from GCR for the case $\alpha = 3$ and $m_g = 0$. The oblique straight line is the border of (12) with k replaced by p, and GCR does not occur in the upper region effectively. The elliptic curve is the border of (15), inside of which (dark-gray region) satisfies (15), where the GCR effect is so small that the extremely high energy cosmic rays do not damp. The light-gray region is excluded by GCR.

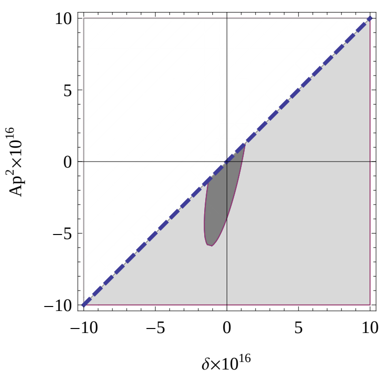

FIG. 3. Same as Fig. 2, but for the case $\alpha = 4$.

high energy cosmic ray with initial energy p during time t as

$$\frac{dE}{dt} \leq \frac{p}{t}, \tag{14}$$

which gives

$$\delta^2 p^4 + \frac{6A^2 p^{2\alpha}}{\alpha(\alpha-1)(2\alpha-1)} - \frac{48\delta A p^{\alpha+2}}{\alpha(\alpha+1)(\alpha+2)} \leq \frac{12p}{G_N t}. \tag{15}$$

We examine conditions (12) and (15). Here we assume that the extremely high energy cosmic rays of $p = 10^{11}$ GeV from a cosmological distance $ct = 1$ Mpc are not significantly affected by the damping through GCR [16]. Figures 2 and 3 show the constraint on the model parameters from GCR for cases $\alpha = 3$ and 4, respectively. The oblique straight dashed line in each figure shows the border of condition (12) with k replaced by p, above which GCR does not arise effectively. GCR arises in the lower region of this straight line. The elliptic regions in Figs. 2 and 3 satisfy condition (15), where GCR arises but the effect is small. The lower light-gray region in each figure is excluded by the GCR process.

For the case $\delta = 0$, i.e., $c_s = 1$, (15) yields

$$|A| \leq 4 \times 10^{5-11\alpha} \sqrt{\alpha(\alpha-1)(2\alpha-1)} \times \left(\frac{10^{11}\ \mathrm{GeV}}{p}\right)^{\alpha-1/2} \left(\frac{1\ \mathrm{Mpc}}{ct}\right)^{\frac{1}{2}} \mathrm{GeV}^{2-\alpha}. \tag{16}$$

Thus, for the case $c_s = 1$, by combining (12) and (15), the constraint from GCR excludes the value

$$Ap \leq -2 \times 10^{-16} \left(\frac{10^{11}\ \mathrm{GeV}}{p}\right)^{3/2} \left(\frac{1\ \mathrm{Mpc}}{ct}\right)^{\frac{1}{2}} \tag{17}$$

for $\alpha = 3$ and

$$Ap^2 \leq -4 \times 10^{-16} \left(\frac{10^{11}\ \mathrm{GeV}}{p}\right)^{3/2} \left(\frac{1\ \mathrm{Mpc}}{ct}\right)^{\frac{1}{2}} \tag{18}$$

for $\alpha = 4$, respectively.

Note that GCR excludes the negative region of the parameter of A when $\delta = 0$, i.e., $c_s = 1$. When $\delta \neq 0$, the constraint on A depends on δ. When $\delta > 0$, a positive region of A is excluded depending on the value of δ.

Now we discuss the implication of our results for specific models with the modified dispersion relation. First, we consider the model in broken-symmetry scenarios [19, 20], in which Lorentz invariance is broken associated with the Planck energy scale:

$$\omega_k^2 = k^2 + m_g^2 + \eta \frac{\omega_k}{E_p} k^2, \tag{19}$$

where E_p is the Planck energy scale and η is a nondimensional parameter. This model corresponds to our model parameters $\alpha = 3$ and $A = \eta/E_p$ and $\delta = 0$. From (17), the following region is excluded:

$$\eta \lesssim -2 \times 10^{-8} \left(\frac{10^{11}\ \mathrm{GeV}}{p}\right)^{5/2} \left(\frac{1\ \mathrm{Mpc}}{ct}\right)^{1/2}. \tag{20}$$

The authors of Ref. [19] argue that η naturally takes a negative value in the broken-symmetry scenarios, and so GCR puts a useful constraint on this scenario. In the previous work in Ref. [21], a constraint similar to ours is obtained for the photon's modified dispersion relation, by comparing synchrotron radiation in the presence of the modified dispersion relation and observations from the Crab Nebula.

We next consider the scenario with an extra dimension [22], which is an example of the case of $\alpha = 4$:

$$\omega_k^2 = k^2 + m_g^2 - \alpha_{\mathrm{edt}} \frac{\omega_k^4}{E_p^2}, \tag{21}$$

where α_{edt} is a nondimensional parameter. This model corresponds to our model parameters $\alpha = 4$ and $A = -\alpha_{\mathrm{edt}}/E_p^2$ and $\delta = 0$. The constraint from GCR (18) excludes the value

$$\alpha_{\mathrm{edt}} \gtrsim 6 \left(\frac{10^{11}\ \mathrm{GeV}}{p}\right)^{7/2} \left(\frac{1\ \mathrm{Mpc}}{ct}\right)^{1/2}. \tag{22}$$

GCR constrains a positive value of $\alpha_{\rm edt}$; otherwise, GCR does not occur. In this scenario of the modified dispersion relation, the sign of $\alpha_{\rm edt}$ is not specified. The authors of Ref. [22] argue that the case $\alpha_{\rm edt} < 0$ is relevant to theories in which a generalized uncertainty principle is used. Therefore, unfortunately, our results cannot resolve this argument.

As discussed in Ref. [12], there are other possible models that predict the modified dispersion relation. For example, the Horava-Lifshitz gravity model predicts a modified dispersion relation [23–27]:

$$\omega_k^2 = c_s^2 k^2 + \frac{\kappa^4 k^4}{16}\left(\mu \pm \frac{2k}{\varpi^2}\right)^2 - \frac{\kappa^2\eta}{2}k^6, \tag{23}$$

for the left-right polarization modes, where κ, μ, ϖ, and η are parameters. The condition (7) at which GCR arises can be generalized as follows:

$$\delta - \frac{\kappa^4 k^2}{16}\left(\mu \pm \frac{2k}{\varpi^2}\right)^2 + \frac{\kappa^2\eta}{2}k^4 \geq 0. \tag{24}$$

Following the theory with the detailed-balance condition, $\eta = 0$, the condition (24) is not satisfied as long as $\delta \leq 0$, which does not allow GCR process. For the theory without the detailed-balance with $\eta > 0$, the condition (24) might be satisfied even when $\delta = 0$. For simplicity, we consider the case $\mu = 0$, which reduces to an example of the case $\alpha = 6$ with $A = \kappa^4/4\varpi^4 - \kappa^2\eta/2$. From (15), we have the constraint on δ and A. For the case $\delta = 0$, GCR excludes the value $A(= \kappa^4/4\varpi^4 - \kappa^2\eta/2) < -7 \times 10^{-60}\mathrm{GeV}^{-4}$, by adopting $p = 10^{11}$ GeV and $ct = 1$ Mpc.

The theory with noncommutative geometries, discussed in Ref. [28], predicts a modified dispersion relation of the form (1) with $\alpha = 4$ at the lowest order of perturbative expansion of the momentum. The sign of the parameter A is naturally predicted to be positive. In this case, GCR does not appear as long as $c_s = 1$.

In Ref. [12], future prospects of constraining the modified dispersion relation from direct measurements of gravitational waves from binary systems with the Advanced LIGO Project are addressed. With the method proposed, it is possible to put a stringent constraint on the graviton mass m_g. For example, for the case $\alpha = 3$, the authors of Ref. [12] predict that the constraints at a level of $m_g < 10^{-22}$–10^{-25} eV will be obtained, depending on the target and the signal-to-noise ratio. However, their constraint on A is not stringent. The best constraint is $|A| < 10\ \mathrm{GeV}^{-1}$ for the case $\alpha = 3$. However, the corresponding constraint on A from GCR is quite stringent, $A \gtrsim -10^{-27}\ \mathrm{GeV}^{-1}$, although the constraint on m_g is not very stringent. Thus, the two methods are complementary.

We have investigated the constraint on the modified dispersion relation for gravitational waves from GCR, assuming the form (1). The constraint on m_g is not very tight; however, the constraint on A from GCR can be very stringent. When the propagation speed is less than the velocity of light, $c_s < 1$, GCR may appear even when A is positive, which is constrained through the GCR process. When $c_s = 1$, GCR puts a constraint on A only when A is negative. This constraint is very stringent compared with that from direct measurements of gravitational waves from binary systems as found in Ref. [12] (see also [13, 14]). Thus, the constraint from GCR is complementary to that from direct measurements of gravitational waves.

After this paper was nearly completed, Ref. [29], in which a similar problem is investigated, appeared on the arXiv. We focused on the modified dispersion relation of the form (1) but the authors of Ref. [29] investigate constraints on various Lorentz violation operators.

We thank R. Kimura, C. Yoo, and D. Blas for useful discussions and comments. The research by KY is supported by a Grant-in-Aid for Scientific Research from the Japan Ministry of Education, Culture, Sports, Science and Technology (No. 15H05895).

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